Finite Space Construction

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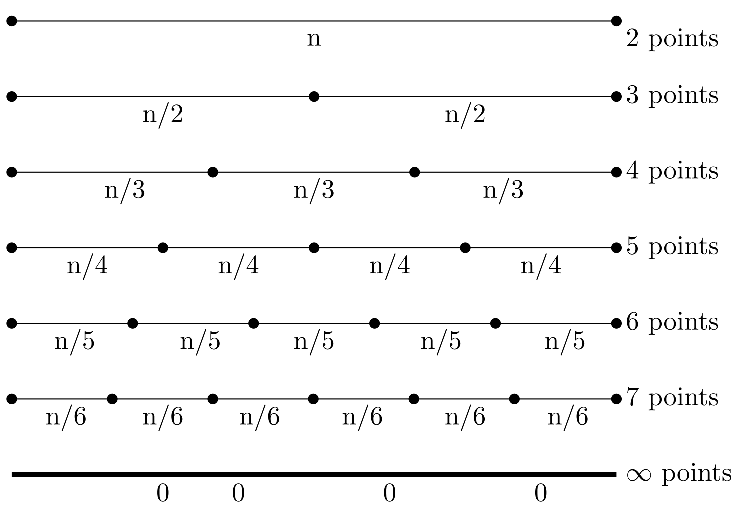
1 Introduction

The former teachings of mathematical spaces taught us that spaces can be both finite and infinite. For example, we are told that the real number line ($\mathbb{R}$) extends infinitely while constituting an infinite space. This paper says that a mathematical space intending to represent physical space is constructed by indivisible units, a mathematical point in space's context; hence, finite spaces of all dimensions are mathematically well defined and form based on a constructive mathematical method, whereas infinite spaces proved to be mathematically undefined and therefore cannot form based on the same method. Hence, this paper is going to show you how an infinite number of points constitutes a finite space and that you cannot formally construct an infinite space from an infinite number of points by means of mathematical methods.

2 Proof



As seen in the illustration above, as you keep adding points, the gaps will become equally smaller but will never become zero in length. However, you need to make the gaps zero in order to construct a continuous space, and the only way to do this is to use a limit that reaches infinity. But you first need a function for that.

We divide the line into ever equally smaller pieces until the gaps between the points become infinitely smaller (exactly like the rectangles in the Riemann sums) to fully close the gaps between all consecutive points to construct a finite continuous space; therefore, we use the limit:

$$\lim_{x\to\infty} \frac{n}{x} = 0$$

The above statement is a function that models the behavior in the illustration and tells us that gaps are incrementally and equally getting smaller until they have zero length. That is, as the number of gaps between points reaches infinity while becoming infinitely smaller due to the finite length, the number of points that surround them reaches infinity while forming a continuous finite space.

3 Results

We can show that, unlike a finite space, an infinite space cannot be formally constructed using the method, that is, if the length of the line reaches infinity as well:

$$\lim_{\substack{n\to\infty \\ x\to\infty}} \frac{n}{x} = \frac{\infty}{\infty}$$

which is an indeterminate form, which, in its current form, is not solvable because n and x are not defined in terms of each other —they are unrelated. The two variables n and x, representing the length of the line and the number of points evenly distributed on the line, respectively, do not have a direct physical relationship. Hence, the limit does not exist and the result is not defined. However, when n is a constant, which means that the length is finite, the equation is well defined and equal to zero, which means that all the gaps between the points are fully closed while forming a continuous finite space. **The real number line must be finite because it is both dense and complete, the real number line cannot be simultaneously infinite, dense, and complete; a fully realized, continuous, gap-free line can only achieve this state within a finite bound.** 2-D and 3-D spaces spaces form in the same way a line forms from indivisible units. Therefore, we can conclude that the outer space is too finite as we perceive it because the outer space is also composed of indivisible units and is both complete and dense.